

# **Innovative Fusion of Experiment and Analysis for Missile Design and Flight Simulation**

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# Introduction

- Integration of experimental and computational data
  - key to supporting decisions during the development of aerospace products
- Heterogeneous data sets
  - Mutually enhanced
  - Multidimensional response surface technology
- Application:
  - Use of sparse experimental data to correct a computational database for use in comprehensive flight simulations of missiles

# Motivation

- Develop global understanding of the data
- Common situation...
- Data fusion technique via response surface methods
  - not conventional interpolation / data fitting
  - computational (model) based
- Dual aspects:
  - interpolation/extrapolation of limited experimental data
  - fine-tuning / calibration of computational models

# Mutual Enhancement of Data Sets

- Data generalization
  - ill-posed problem
  - regularizing assumptions
    - physics based models
    - mathematical equations
    - smoothness assumptions
    - empiricism
    - **Hypersurface (NEAR RS)**
      - goes through the experimental data
      - “supported” by additional computational constraints

# $N$ -Dimensional Response Surface Calculation

- Identify smooth mapping  $F: \mathbb{R}^N \rightarrow \mathbb{R}$   $F(\mathbf{X}_i) = Y_i, \quad i = 1, \dots, p$
- Minimize the distance  $\| F(X_i) - Y_i \|^2$
- Expand  $F$  into radial basis functions

$$F(X) = \sum_k c_k \phi_k(X), \quad \phi_k(X) = f(\| X - \chi_k \|^2; b)$$

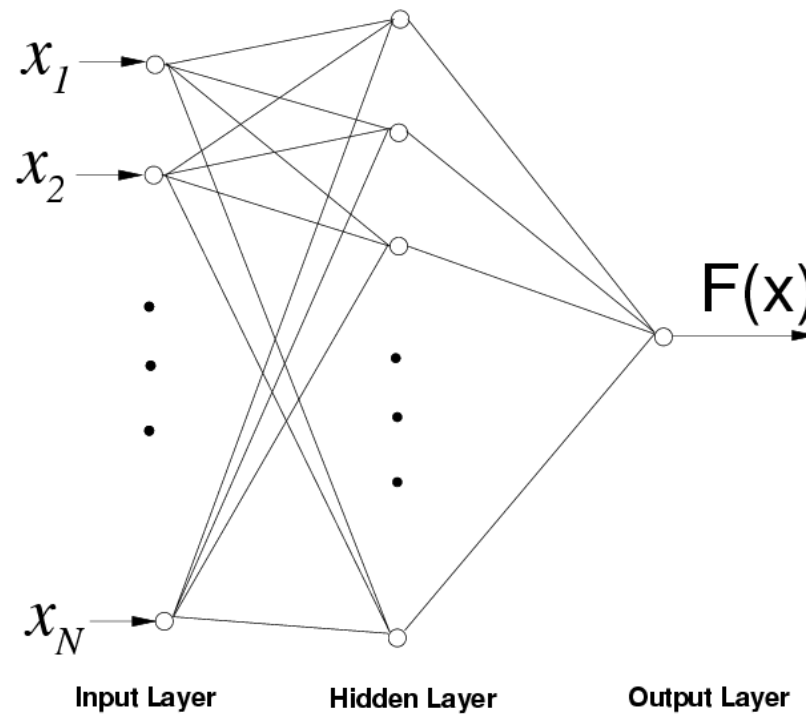
$f$  is a shape function

$b$  is a scale or stiffness parameter

$(\chi_k; F(\chi_k))$  RS support vectors

$c_k$  solution of least-squares problem

# Radial Basis Function Network

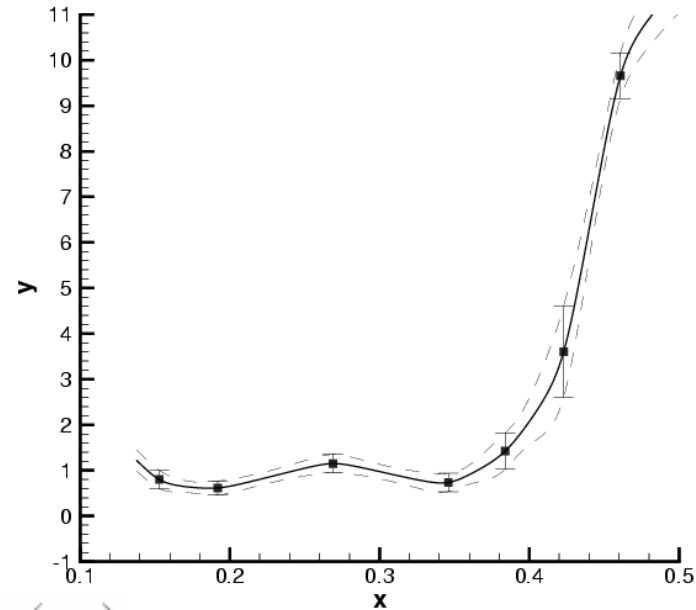


# Uncertainty Prediction (NEAR RS)

- If  $f$ ,  $b$ ,  $\chi_k$  are known  $\Rightarrow$  linear system  $[A][C] = [Y]$
- Regressor model for the data:  $[A][C] = [Y] + [e]$
- Since  $F(X) = \sum_k c_k \phi_k(X)$
- Propagate uncertainty:

$$\text{var}(F) = \sum_i \sum_j (\text{cov}[C])_{ij} \phi_i(X) \phi_j(X)$$

$$\delta Y_i \rightarrow \delta c_k \rightarrow \delta F$$



# Application to Data Fusion

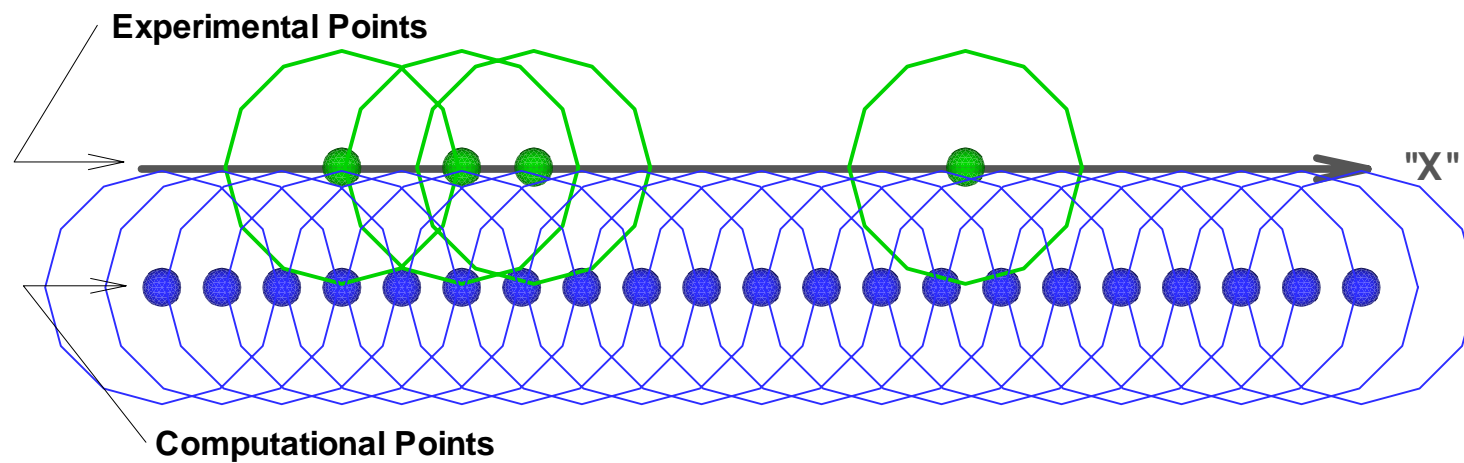
- “The combination of a group of inputs with the objective of producing a single output of greater quality and greater reliability.” (Li et al., 1993)
- Two sources of data
  - computational (approximations in physical models)
  - experimental (limitations, cost, sparse)
  - not same sampling, conditions
- Construct global metamodel incorporating all the data



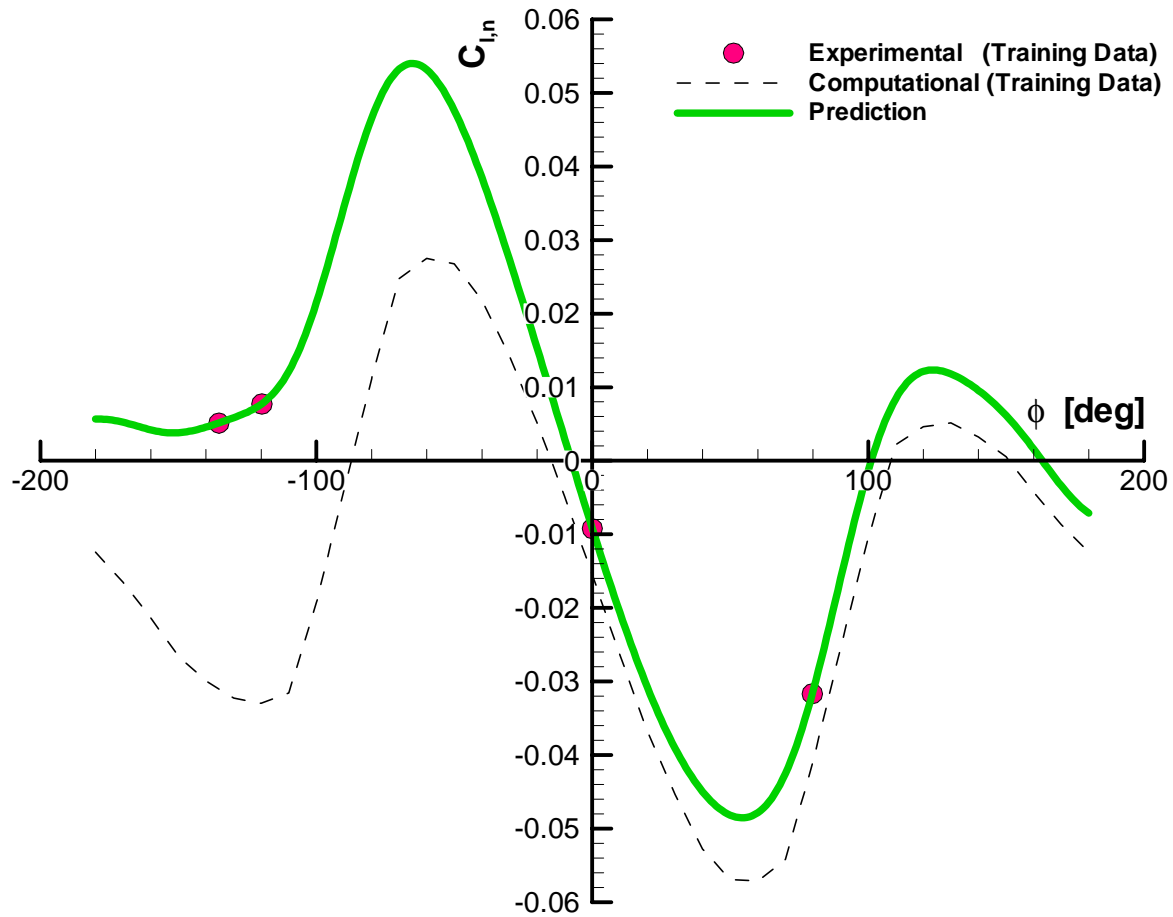
## Method

- Introduce auxiliary variable  $\varepsilon \equiv x_{N+1}$  added to the multidimensional space  $(x_1, x_2, \dots, x_N)$
- $\varepsilon$  used to tag whether data are computational  $\varepsilon = 0$  or experimental  $\varepsilon = 1$
- Single response surface calculated in  $N+1$  dimensions, queried in the  $\varepsilon = 1$  subspace

# Geometric Interpretation



# Fusion Example



## Fusion Example (Concluded)

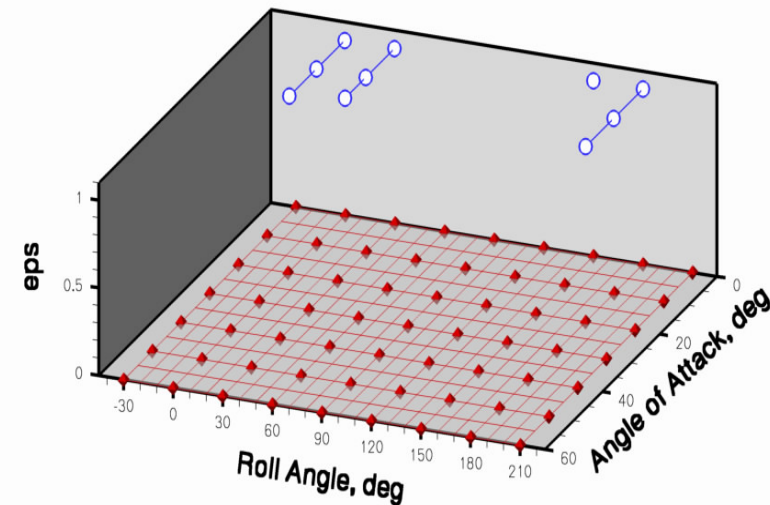
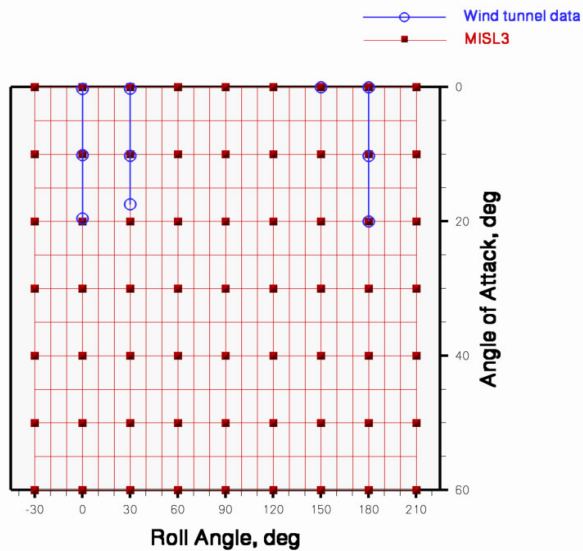
- Importance of using computational models which incorporate the correct physics
  - specially important when performing sparse data interpolation/extrapolation
- “Two experimental points” example is relevant to the case of multidimensional data
  - finite resources (time and budget) limit the number of conditions that can be acquired
  - modern design-of-experiment techniques can help, but
  - “filling in” the space remains an impossibility when the number of independent variables is large
  - sparse sampling in *some* directions to be expected

# Correction of Aerodynamic Databases Using Experimental Data

- Assimilation of limited wind tunnel data
  - goal: increase the accuracy of comprehensive flight simulations of a missile
- Generic body-tail configuration
- Two data sets
  - sparse experimental (wind tunnel) data
  - “computational” database (MISL3)
    - Forces and moments
    - Wide range of angles of attack, roll angles, and Mach numbers
- “Error database”

# Error database

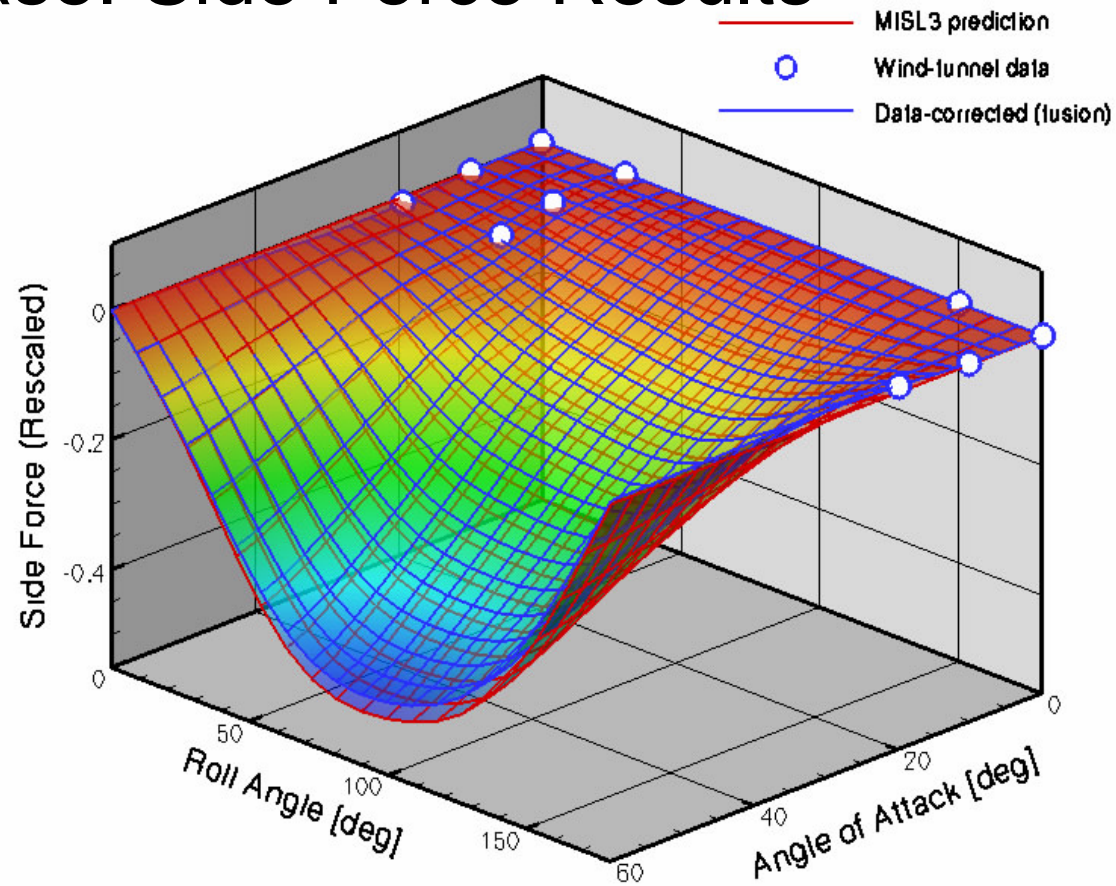
- Defined as difference between two fits
  - Four-dimensional  $F(\alpha_c, \varphi, M, \varepsilon)$
  - Analytic (smoothly varying)



## Error database (Cont'd)

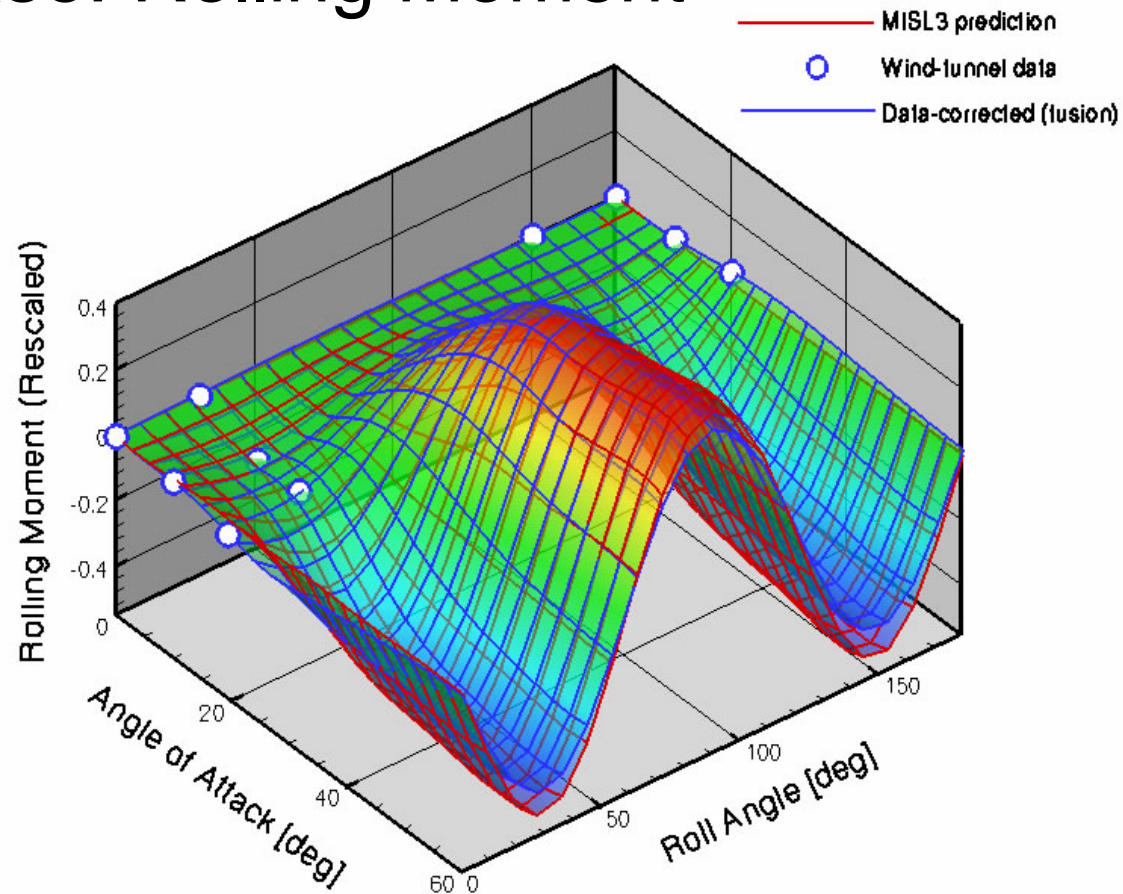
- Used to “correct” MISL3 database
  - takes into account experimental measurements
- Smart interpolation/extrapolation
  - process is automatic
  - no equations specified
  - requires only the specification of support vectors

# Wind Tunnel Data Enhancement of MISL3 Database: Side Force Results



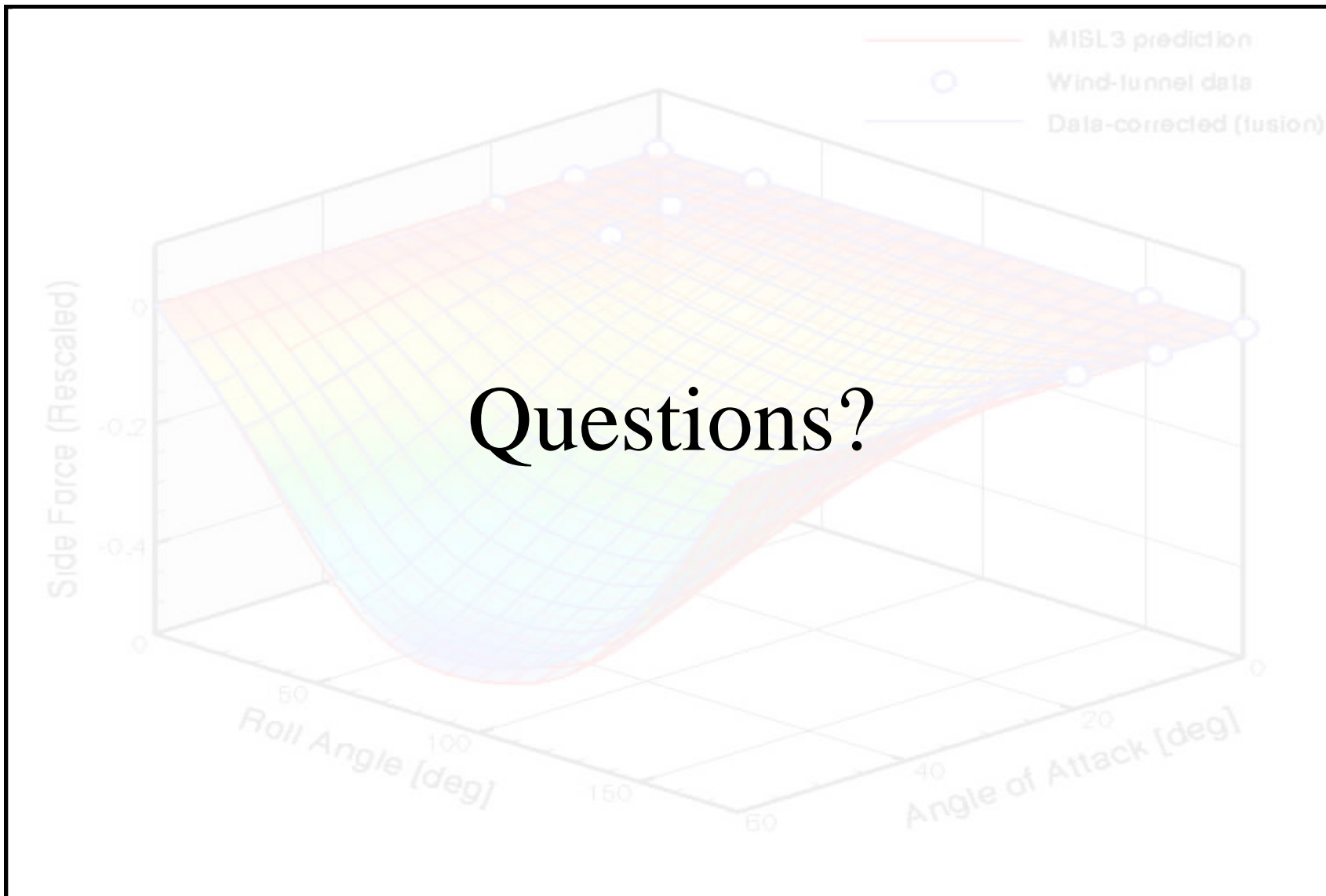


# Wind Tunnel Data Enhancement of MISL3 Database: Rolling Moment



# Conclusions

- Fusion of experimental and computational data via dimensionality augmentation and RS methods
- Fully analytic, mathematical description
  - easy to use (support vector specification)
  - data structure flexibility / use of heterogeneous data sets
  - rational basis for propagating uncertainty estimates
- Assimilation of limited wind tunnel data with computational databases
  - construct smart interpolation and extrapolation schemes
  - fine-tune the results of computational analyses



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